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CRITICAL COMPARISON OF THE COLLOCATED AND STAGGERED GRID ARRANGEMENTS FOR INCOMPRESSIBLE TURBULENT FLOWS

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Abstract. The collocated-mesh scheme is often favored over the staggered-mesh scheme for turbulence simulation in complex geometries due to its slightly simpler form in curvilinear coordinates. The collocated mesh scheme does not conserve kinetic energy however, and few careful checks of the impact of these errors have been made. In this work, analysis is used to identify two sources of kinetic energy conservation error in the collocated-mesh scheme: (1) interpolation errors arising from second-order linear interpolation and (2) pressure errors. It is shown that the interpolation error can be eliminated through the use of a first-order accurate centered interpolation operator with mesh-independent weights. The pressure error can not be eliminated and it is shown to scale as $O(\Delta t^2 \Delta x^2)$. The effects of the conservation errors is investigated numerically by performing simulations of turbulent channel flow as well as inviscid simulations of the flow over an airfoil. The channel flow results are compared with those of a staggered-mesh code. Neither the second-order interpolation error nor the pressure error appear to lead to significant error in the channel where the Cartesian mesh is stretched in only one direction. The airfoil simulations performed in curvilinear coordinates show a much greater sensitivity to the interpolation errors. The second-order centered interpolation lead to severe numerical oscillations, while the kinetic energy-conserving first-order centered interpolation produce solutions that are almost as smooth as those obtained with a second-order upwind interpolation. These results suggest that numerical oscillations can be controlled in curvilinear coordinates through the use of properly-constructed non-dissipative centered interpolations.

1. Introduction

Numerical Simulation of turbulent flows, using either Direct Numerical Simulation (DNS) or Large Eddy Simulation (LES), requires high-fidelity numerical methods. For incompressible flow, it is highly desirable to have a scheme that conserves mass, momentum, and kinetic energy. In practice, it is rather difficult to satisfy these three constraints simultaneously and one is often faced with the need to give up strict conservation. For computations in Cartesian coordinates, the staggered-mesh scheme is most often used since it is fully-conservative in this case. The extension of the scheme to curvilinear coordinates is not entirely straightforward however, and many researchers have opted for simpler formulations. Foremost among these is the so-called collocated-mesh scheme, which has been used by a number of investigators (Peric, 1985), (Peric *et al.*, 1988), (Zang *et al.*, 1994), (Ye *et al.*, 1998), (Armenio and Piomelli, 2000), who were interested in performing LES in complex geometries using body-fitted grids.

Morinishi *et al.* (1998) analyzed the conservation properties of several finite-difference schemes for both staggered and collocated grid arrangements. By restricting the analysis to Cartesian uniform meshes, Morinishi *et al.* showed that staggered-mesh methods can be made fully-conservative, whereas collocated-mesh methods will always contain an energy conservation error of the form $O(\Delta t^m \Delta x^n)$. On curvilinear meshes the collocated-mesh scheme may also develop a second kinetic energy conservation error due to interpolation errors. Like the others before us, we were motivated to use the collocated-mesh scheme for complex flow LES due to its simpler form. Before doing this, however, we wanted to perform analysis and numerical experiments to investigate the impact of the kinetic energy conservation errors. We were also concerned with the prevalence of upwind interpolations used by prior investigators when performing LES with the collocated-mesh scheme in curvilinear coordinates. Any serious problems stemming from the kinetic energy conservation errors, or from the use of upwind interpolations would be grounds for us to reject the collocated-mesh scheme and simply code the staggered-mesh scheme in curvilinear coordinates.

The objective of this paper is twofold: (1) to clearly state the source of the conservation errors through analysis, and (2) to compare the performance of the two schemes for LES of turbulent channel flow, and for the inviscid flow over an airfoil.

2. Analysis

First we define an interpolation operator that approximates the function ϕ at an arbitrary position x^* , which lies between the mesh points x and

$x + \Delta x$

$$\phi(x^*) \simeq \bar{\phi}^x \equiv (1 - w)\phi(x) + w\phi(x + \Delta x). \quad (1)$$

This formula is second order accurate if $w = (x^* - x)/\Delta x$ and is first order accurate otherwise. It is useful to rewrite the interpolation operator as the sum of two operators; one with the mesh-independent weights and a second containing the mesh information:

$$\bar{\phi}^x = \underbrace{\frac{1}{2}[\phi(x) + \phi(x + \Delta x)]}_{\bar{\phi}^{x^0}} + \underbrace{\left(w - \frac{1}{2}\right) \Delta x \frac{\delta\phi}{\delta x}}_r, \quad (2)$$

where $\delta\phi/\delta x = (\phi_{i+1} - \phi_i)/\Delta x$.

In addition, we define a special interpolation operator for the product of ϕ and ψ :

$$\widehat{\phi\psi}^x = \frac{1}{2}\phi(x)\psi_i(x + \Delta x) + \frac{1}{2}\psi(x)\phi(x + \Delta x). \quad (3)$$

The following identity involving these two interpolation operators will be needed later on in the paper

$$\phi \frac{\delta(\psi \cdot \bar{\phi}^{x^0})}{\delta x} = \frac{1}{2} \frac{\delta(\psi \cdot \widehat{\phi\phi}^x)}{\delta x} + \frac{1}{2} \phi\phi \frac{\delta\psi}{\delta x}. \quad (4)$$

2.1. STAGGERED GRID SYSTEM

The staggered mesh arrangement in Cartesian coordinates is shown in figure 1. The velocity components U_i (or U, V, W) are distributed around the pressure points p . This layout has the advantage that, when multiplied by the cell face area, the velocity components give the exact volume fluxes, F_i . This feature leads to a simplified mass balance computation and results in fully-coupled velocity and pressure fields.

2.2. COLLOCATED GRID SYSTEM

The collocated-mesh arrangement in Cartesian coordinates is shown in figure 2. The velocity components u_i (or u, v, w) are stored with the pressure p at the cell center. In addition, volume fluxes, f_i , are defined at the cell face in a manner analogous to the staggered-mesh system. The volume fluxes are not solution variables, but rather are determined through interpolation of the cell-centered u_i values plus a projection operation that guarantees exact conservation of mass (Rhie and Chow, 1983). Use of the

mass-conserving volume fluxes results in a pressure equation identical to that in the staggered-mesh system and thus also leads to fully-coupled velocity and pressure fields. The only drawback of the collocated-mesh scheme is that cell-center values, u_i , are only approximately divergence free. This feature leads to a kinetic energy conservation error as discussed below.

2.3. KINETIC ENERGY CONSERVATION

2.3.1. *Staggered-grid system*

When properly-formulated, the second-order staggered-mesh scheme should conserve mass, momentum, and kinetic energy, irrespective of the underlying coordinate system. In developing higher-order schemes, Morinishi *et al.* (1998) first reviewed the conservation properties of several existing schemes cast in uniform Cartesian coordinates. They were able to show that all correctly-coded second-order accurate forms of the non-linear terms (divergence, advective, rotational, and skew-symmetric) are equivalent numerically and fully-conservative. Later Vasilyev (2000) extended the work of Morinishi *et al.* to the case of non-uniform Cartesian meshes. In this work, Vasilyev advocated the use of a mapping to uniform computational space where grid-independent difference and averaging operators could be used. In spite of this, he chose to analyze the divergence form of the non-linear terms in physical space. He found that such a formulation does not conserve kinetic energy due to a lack of commutativity between the average and difference operators. This error can be dispensed with by choosing to work with the non-linear term written in the uniform computational space. The transformation is quite simple in the case of a non-uniform Cartesian mesh, and the fully-conservative formulation can be written as

$$\frac{\delta F_j}{\delta \xi_j} = 0, \quad (5)$$

$$\frac{\delta U_i}{\delta t} + \frac{1}{V^{\xi_i}} \frac{\delta}{\delta \xi_j} \left(\bar{U}_i^{\xi_j} \bar{F}_j^{\xi_i} \right) + \frac{\delta p}{\delta x_i} + (\text{visc})_i = 0, \quad (6)$$

where V is the cell volume, F_i is the volume flux, ξ_i is a computational space with unit displacements, and $(\text{visc})_i$ are the viscous terms. The commutation error discussed by Vasilyev does not appear in this formulation since both the average and difference operations are performed in the uniform computational space. The only subtle point is that this formulation requires an average of the physical velocity components in the computational space (i.e. $\bar{U}_i^{\xi_j}$). Although this operation is easy to code (weights of 1/2), it results in a an approximation that is only first order accurate. This same issue arises in the collocated mesh scheme and will be discussed in more detail below.

2.3.2. Collocated-grid system

The mass and momentum conservation equations for the collocated-mesh system in curvilinear coordinates are

$$\frac{\delta f_j}{\delta \xi_j} = 0, \quad (7)$$

$$\frac{\delta u_i}{\delta t} + J \frac{\delta}{\delta \xi_j} \left(\bar{u}_i^{\xi_j} f_j \right) + J \frac{\delta}{\delta \xi_j} \left(\overline{J^{-1} \xi_i^j p^{\xi_j}} \right) + (\text{visc})_i = 0, \quad (8)$$

where $\xi_i^j = \frac{\delta \xi_j}{\delta x_i}$ and J^{-1} is the jacobian of the transformation.

In the case of the collocated layout, the pressure term is primarily responsible for the lack of kinetic energy conservation. The convective terms may or may not conserve kinetic energy depending on the details of the interpolation operator. Note that the interpolation $\bar{u}_i^{\xi_j}$ suggests interpolating the physical velocity u_i in the computational space ξ_j . While this is a first order accurate approximation (since it makes no account for the physical distances), it results in a kinetic energy conserving formulation. In order to retain second order accuracy, it is tempting to compute mesh-dependent weighting factors and make use of Eq. (2) for the interpolation. This attempt for higher accuracy actually spoils kinetic energy conservation. To see this, we first write the interpolation operator in the convective terms in the form of Eq. (2) and dot the result with the velocity vector to get

$$u_i \frac{\delta}{\delta \xi_j} \left(\bar{u}_i^{S_j} f_j \right) = u_i \frac{\delta}{\delta \xi_j} \left(\bar{u}_i^{S_j^0} f_j + r_j \frac{\delta u_i}{\delta S_j} f_j \right). \quad (9)$$

where S_j is the physical distance measured along the mesh direction ξ_j . Note the trivial equivalence: $\overline{(\cdot)}^{S_j^0} \equiv \overline{(\cdot)}^{\xi_j}$.

Now making use of Eq. (4), Eq. (9) becomes

$$u_i \frac{\delta}{\delta \xi_j} \left(\bar{u}_i^{S_j} f_j \right) = \frac{1}{2} \frac{\delta}{\delta \xi_j} \left(\widehat{u_i} \widehat{u_i}^{S_j} f_j \right) + \frac{1}{2} u_i^2 \frac{\delta f_j}{\delta \xi_j} + u_i \frac{\delta}{\delta \xi_j} \left(r_j \frac{\delta u_i}{\delta S_j} f_j \right). \quad (10)$$

The first term on the right hand side is in divergence form and is thus conservative. The second term vanishes due to the continuity relation. The final term represents the kinetic energy error arising from the interpolation. This term vanishes when $r_j = 0$, which corresponds to mesh-independent weights of 1/2 in Eq. (2).

The kinetic energy conservation property of the pressure term is analyzed in a similar fashion by starting with the interpolation in the form of

Eq. (2):

$$\begin{aligned}
 u_i \frac{\delta}{\delta \xi_j} \left(\overline{J^{-1} \xi_i^j p}^{S_j} \right) &= u_i \frac{\delta}{\delta \xi_j} \left(\overline{J^{-1} \xi_i^j p}^{S_j^0} \right) + u_i \frac{\delta}{\delta \xi_j} \left(r_j \frac{\delta}{\delta S_j} \left(J^{-1} \xi_i^j p \right) \right), \\
 &= \frac{\delta}{\delta \xi_j} \left(\overline{J^{-1} \xi_i^j p}^{S_j^0} \overline{u_i}^{\xi_j} \right) - p \frac{\delta}{\delta \xi_j} \left(J^{-1} \xi_i^j \overline{u_i}^{\xi_j} \right) + \\
 &\quad u_i \frac{\delta}{\delta \xi_j} \left(r_j \frac{\delta}{\delta S_j} \left(J^{-1} \xi_i^j p \right) \right). \tag{11}
 \end{aligned}$$

Using the Rhie and Chow (1983) interpolation defined as

$$f_j = J^{-1} \xi_i^j \left(\overline{u_i}^{\xi_j} - \Delta t \frac{\delta p}{\delta \xi_j} \delta_{ij} \right)$$

Eq.(11) becomes

$$\begin{aligned}
 u_i \frac{\delta}{\delta \xi_j} \left(\overline{J^{-1} \xi_i^j p}^{S_j} \right) &= \frac{\delta}{\delta \xi_j} \left(\overline{J^{-1} \xi_i^j p}^{S_j^0} \overline{u_i}^{\xi_j} \right) - p \frac{\delta f_j}{\delta \xi_j} - \\
 &\quad p \Delta t \frac{\delta^2}{\delta \xi_i \delta \xi_j} \left(J^{-1} \xi_i^j p \right) + u_i \frac{\delta}{\delta \xi_j} \left(r_j \frac{\delta}{\delta S_j} \left(J^{-1} \xi_i^j p \right) \right) \tag{12}
 \end{aligned}$$

The first term on the right hand side is in divergence form and is thus conservative. The second term vanishes due to the continuity relation. The remaining two terms are the kinetic energy error arising from the fact that the cell-center velocities do not conserve mass exactly. The final term vanishes in the case of a first order interpolation ($r_j = 0$).

The pressure error terms are added to the convective error term to form $(E_{ke})_{coll}$, the total error in kinetic energy conservation for the collocated-mesh arrangement:

$$(E_{ke})_{coll} = u_i \underbrace{\frac{\delta \left(r_j \left(f_j \frac{\delta u_i}{\delta \xi_j} + \frac{\delta (J^{-1} \xi_i^j p)}{\delta S_j} \right) \right)}{\delta \xi_j}}_{\text{Interpolation Errors}} - p \Delta t \underbrace{\frac{\delta^2 (J^{-1} \xi_i^j p)}{\delta \xi_j \delta \xi_i}}_{\text{Pressure Error}}. \tag{13}$$

This analysis shows that there are two sources of kinetic energy conservation error for the collocated-mesh scheme. The interpolation error will be present if second order, mesh-dependent weighting factors are used. It can be eliminated by choosing first order fixed weights of 1/2 ($r_j = 0$). Veldman and Verstappen (1992), (1998) recognized the conservation error associated with mesh-dependent averaging weights and opted for constant weights of

$1/2$, in the case of a non-transformed staggered mesh system. Although the pressure error can not be eliminated, it can be reduced to $O(\Delta t^2)$ through the use of the Van Kan scheme (van Kan, 1986). In this formulation, one effectively projects with $\delta p = p^{n+1} - p^n \simeq (\partial p / \partial t) \Delta t$ instead of p in Eq. (12) and thus the terms proportional to p in Eq. (13) are reduced by a factor of Δt .

There are two important questions regarding the kinetic energy errors: (1) are the pressure errors strong enough to de-stabilize the scheme, and (2) does the kinetic energy violation due to second order interpolations negate any increase in accuracy over the first order (kinetic energy conserving) interpolations? We will explore these questions in the following section where the numerical experiments are discussed.

3. Numerical results

3.1. TURBULENT CHANNEL FLOW

The influence of the two sources of kinetic energy conservation error are evaluated through LES of plane channel flow at $Re_\tau = 400$, based on the channel half width and friction velocity. Two computer codes are used; one is based on the second-order staggered-mesh arrangement in the form of Eq. (6) and the other is based on the second-order collocated-mesh arrangement. In either case, finite differences are used only in the streamwise and wall-normal directions and Fourier collocation is used in the spanwise direction. This arrangement allows for more efficient convergence studies since it is only necessary to vary the mesh spacings in the x and y directions in order to investigate the effects of the numerical error. Both codes make use of a third-order Runge-Kutta explicit time marching scheme. The spanwise direction is de-aliased at no computational expense through the use of mesh shifting (Rogallo, 1981). This is done in concert with the multi-step Runge-Kutta scheme. The pressure Poisson equation is solved directly via Fourier transforms in x and z and tri-diagonal inversion in y direction.

Three mesh resolutions and several time step sizes are investigated in order to study the effect of the discretization and the time stepping errors (see Table 1). The computational domain is $8\delta \times 2\delta \times 2\delta$, where δ is the channel half width. The subgrid-scale stresses are modeled using the Smagorinsky (1963) model in conjunction with a Van-Driest type wall-damping function. The results are compared with the DNS data of Moser *et al.* (1999) for $Re_\tau = 395$. For the convergence studies, the time step was held fixed for the three mesh resolutions. This time step corresponds to a viscous $CFL = 1.5$ on mesh C. Cases were also run at larger time steps for meshes A and B.

Figure 3 shows the mean velocity profiles for the three mesh resolutions.

As expected, both the staggered and collocated results improve as the resolution is increased. The staggered-mesh results are consistently closer to the DNS data at all resolutions, and a very good agreement is achieved in the sublayer and log region on the finest mesh (case C). The collocated-mesh results are also reasonably accurate on mesh C.

Since the collocated mesh scheme in conjunction with the Van Kan scheme (van Kan, 1986) has a kinetic energy conservation error that scales like Δt^2 , we investigated the possibility that the differences in Figure 3 are from this source. The time step was reduced by 50% on mesh C, and increased by 400% on mesh B. In either case, the results were almost invariant to changes in the time step. This leads us to believe that the energy conservation errors in the collocated mesh scheme do not have a visible impact on the results.

Figure 4 shows a comparison of the turbulent velocity fluctuations. For the sake of clarity, only results for u_{rms} and v_{rms} , for cases A and C are shown. Once again the staggered-mesh results are superior to the collocated-mesh at all resolutions. The DNS data were not filtered and thus some of the apparent difference between the LES and DNS is due to the unresolved energy in the LES. This effect should be minimal on mesh C. The differences between the two schemes on the finest mesh (C) are minor and either method produces reasonable results at this resolution. The velocity fluctuation profiles are also rather insensitive to changes in the time step.

The rate of convergence of the LES results to the DNS data was investigated by forming the rms difference between the LES and DNS mean velocity profiles:

$$E_{rms} = \sqrt{\frac{1}{2\delta} \int_0^{2\delta} (U_{LES} - U_{DNS})^2 dy} \quad (14)$$

Figure 5 shows the E_{rms} for both mesh arrangements as a function of the grid resolution. The two grid arrangements behave similarly, showing something close to second order convergence between the finest two meshes. The Collocated mesh scheme appears to converge at a slightly higher rate, which is consistent with the generally poorer results on the coarser meshes. The collocated mesh scheme also has a slightly smaller rms error as compared with the staggered mesh scheme, even though Figure 3 would suggest the opposite. The reason for this is that Figure 3 is plotted on a log scale, which emphasizes the near-wall region.

The convergence results suggest that the first order momentum interpolations do not have a visible impact on the convergence rate at these resolutions. This may be due to the fact that the interpolation error is multiplied by the factor $(w - 1/2)$ where w is the averaging weight required

for second order accuracy. For relatively coarse, but smooth meshes, the difference ($w - 1/2$) is of the order of the mesh spacing, and thus one will not see the effect of the first-order interpolation until the mesh is refined further.

We also investigated the use of the second order interpolation weights. The results were similar to those shown in Figures 3 and 4, but with a general small degradation in accuracy. We attribute the degradation to the kinetic energy conservation error. More exacting tests of the interpolation scheme will be presented in the following section where the flow around an airfoil is simulated on a curvilinear mesh.

3.2. INVISCID FLOW AROUND AN AIRFOIL

The inviscid two-dimensional flow over a NACA-0012 airfoil at zero angle of attack has been considered to test the behavior of the collocated mesh arrangement in curvilinear coordinates. An O-mesh containing 50×30 grid-points with the outer boundary placed at four chords is used for the study. The mesh is stretched in both the radial and azimuthal directions in order to better resolve the flow near the leading and trailing edges.

The simulation are performed using three different momentum interpolation operators: (1) the first-order centered interpolation ($w = 1/2$), (2) the second-order centered interpolation ($w = s^*/\Delta s$), and (3) the second-order upwind interpolation (QUICK-type (Leonard, 1979)).

Figure 6 shows the distribution of the pressure coefficient along the airfoil for the three operators considered. The results for the first order centered and second order upwind interpolations are in good agreement with the results of a potential flow analysis. The results for the second-order centered interpolation, on the other hand, show strong oscillations. More insight can be gained from a comparison of the streamwise velocity contours shown in Figure 7. As expected, the dissipative second-order upwind interpolation gives the smoothest solution. On the opposite extreme, the second-order centered interpolation produces a flow field that is obscured by numerical oscillations. The first-order centered interpolation, on the other hand gives a reasonably smooth solution that is comparable to the upwind interpolation. These results are rather significant since they imply that numerical oscillations can be controlled without resort to dissipative upwind schemes. This is particularly important for turbulence simulation where the effects of dissipation have a large negative impact on the solution.

4. Conclusions

We have shown that, in general, the collocated-mesh scheme violates kinetic energy conservation due to two sources: (1) interpolation errors and

(2) pressure errors. The first of these can be eliminated through the use of mesh-independent centered interpolation operators. Although these operators are formally only first order accurate, they are multiplied by geometric terms that are of the order of the mesh spacing for practical computations. We observed second order convergence under mesh refinement for typical LES meshes used in turbulent channel flow. The kinetic energy conservation errors associated with second-order centered interpolations were shown to lead to fairly severe numerical oscillations in a test case involving the inviscid flow around an airfoil computed in curvilinear coordinates. The oscillations could be controlled either by switching to a kinetic energy-conserving first-order centered interpolation, or by switching to a second-order upwind interpolation. We favor the centered interpolation approach for turbulence simulation since it is non-dissipative. This work suggests that the use of upwind interpolations may be unnecessary when the collocated-mesh scheme is applied in curvilinear coordinates, which would significantly increase the fidelity of the numerical method for LES applications.

It does not appear possible to eliminate the second source of kinetic energy conservation error (pressure error). It is possible, however, to limit the size of this error to $O(\Delta t^2 \Delta x^2)$. We could not discern any evidence of this error in turbulent channel flow simulations, where its magnitude was varied by a factor of 16 through time step refinement. These results suggest that pressure term is probably not a serious issue for LES where reasonably fine meshes and small time steps are required for general accuracy purposes.

Our overall conclusion is that the collocated-mesh scheme behaves rather similar to the staggered-mesh scheme, provided that the correct interpolation operators are used. Given its slightly simpler form in curvilinear coordinates, the collocated-mesh scheme may be a better overall choice for LES in complex geometries.

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TABLE 1. Mesh spacings used for the LES of turbulent plane channel flow.

Case	N_x	N_y	N_z	Δx^+	Δy^+	Δz^+	$\Delta t \cdot u_\tau / \delta$
A	16	16	32	200	4~127.15	25	1.36×10^{-3}
B	32	32	32	100	2~62.63	25	1.36×10^{-3}
C	64	64	32	50	1~30.91	25	1.36×10^{-3}

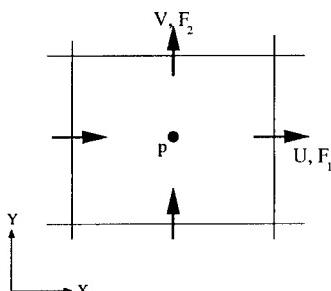


Figure 1. Staggered mesh arrangement in two-dimensional plane.

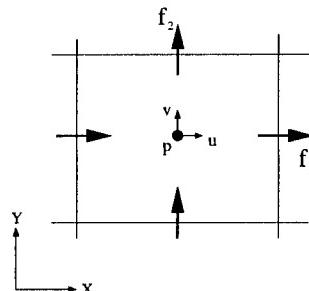


Figure 2. Collocated mesh arrangement in two-dimensional plane.

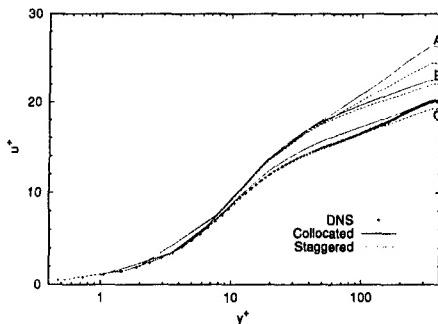


Figure 3. Convergence of the mean velocity profile.

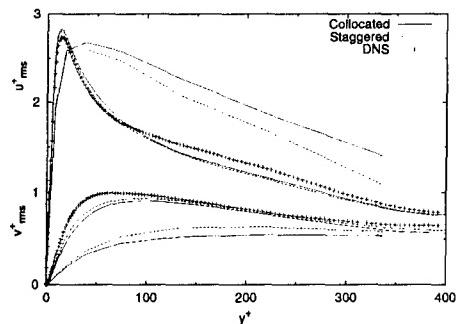


Figure 4. Convergence of the velocity fluctuations.

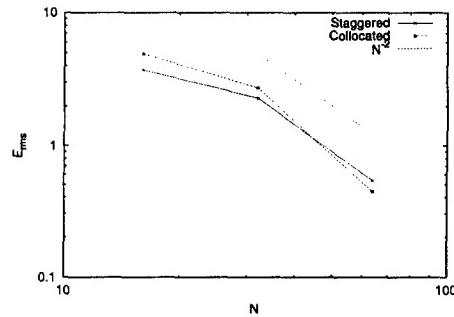


Figure 5. Convergence history as a function of the mesh resolution.

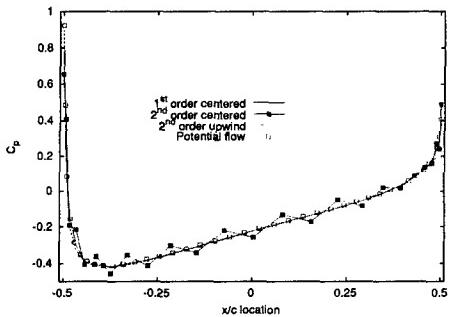


Figure 6. Inviscid NACA-0012 airfoil flow: pressure coefficient distribution.

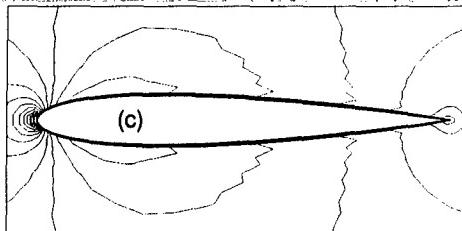
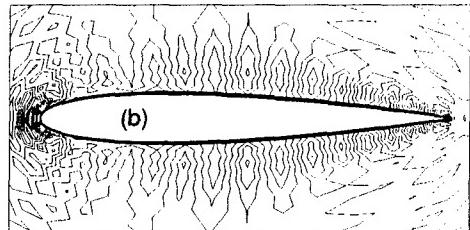
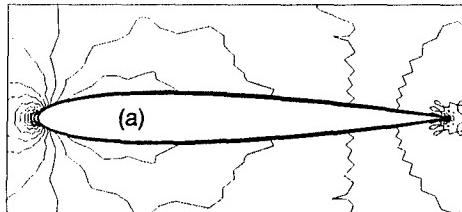


Figure 7. Streamwise velocity contours for the airfoil flow. (a) First-order centered interpolation, (b) Second-order centered interpolation, and (c) Second-order upwind interpolation.